Math of Business Cards and Paper Airplanes

By Scott Little

(Originally written in November 2006). In addition to being a member of Society for Amateur Scientists (SAS), I am also a member of the Society for Industrial and Applied Mathematics (SIAM). While reading this month's newsletter, I came across an interesting article on the properties of falling paper using business cards (1). The article is based on the research of mathematician Z. Jane Wang of Cornell University. Dr. Wang has done extensive research on how insects actually fly, and takes a standard sized business card, 2-1/2"x3" and drops it from an elevation to demonstrate the random movements of an insect wing. By using a single card, Dr. Wang is, in her own words "setting the wing free". Then Dr. Wang can study its aerodynamic properties.

Most objects that can fly, including airplanes, do so by taking advantage of the Bernoulli Principle. The principle states, in simplified terms, that a fall in pressure of a fluid will always be accompanied by an increase in speed (2). In the case of an airplane, the fluid is the air, and the drop in pressure is caused by the difference in speed of the air moving over the top and bottom of the wing. The air takes the same amount of time for it to travel over the top or bottom of the wing, but the distance is longer for the top due the wing's curvature. This creates a drop in the pressure over the top of the wing, and the higher pressure underneath pushes up on the bottom of the wing, causing lift. (Please see figure 3 for an illustration of this effect.)

Mathematically, the Bernoulli Principle is represented by the following equation:

P+(KE/V)=C

Where:
P=pressure
KE= kinetic(moving) energy of the object
V= volume of fluid
C=constant

In the case of an airplane wing, the volume is normally calculated to be the cubic feet of air under the wings. Although this can explain many types of flight, it does not account for the random flight of certain insects. Dr. Wang believes that the same physical forces that explain how creatures such as the Bumble Bee are able to fly also explains the behavior of falling business cards. Dr. Wang uses a set of equations known as the Napier-Stokes Equations to explain the movements of these fluttering objects. The Napier-Stokes Equations are a set of differential equations that are used to calculate the flow of a fluid around an object. In its most basic form, the general equation is (3):

$$\rho \, Dv/Dt = -\nabla p + \nabla \cdot T + \mathbf{f}$$

The left side is the pressure of the fluid times the change in its' speed over time.

The right side of the equation is the summation of all the forces acting on the body.

$-\nabla p$= the pressure gradient, that is, the sum of the dimensional pressure forces from the normal stresses in the fluid flow. These dimensional forces are in the X, Y, Z directions and sometimes include time "t".

$\nabla \cdot T$= the shear forces of the fluid. These are forces acting at an angle to the pressure and could include cross winds, etc.

f= the other forces, including gravity.

As can be seen from the above explanation, the pressure is affected by the speed, time, shear forces, and other forces, such as gravity, to explain lift. To do more precise measurements, Dr. Wang developed some specialized aluminum plates the same size as a business card and then dropped these plates in a tank full of water to film their movements with a high-speed camera.

For my own experiment, I went back to the basics and used the standard paper business card and dropped them off my stairway. I then tracked their approximate movements and distance traveled and plotted them onto a diagram.

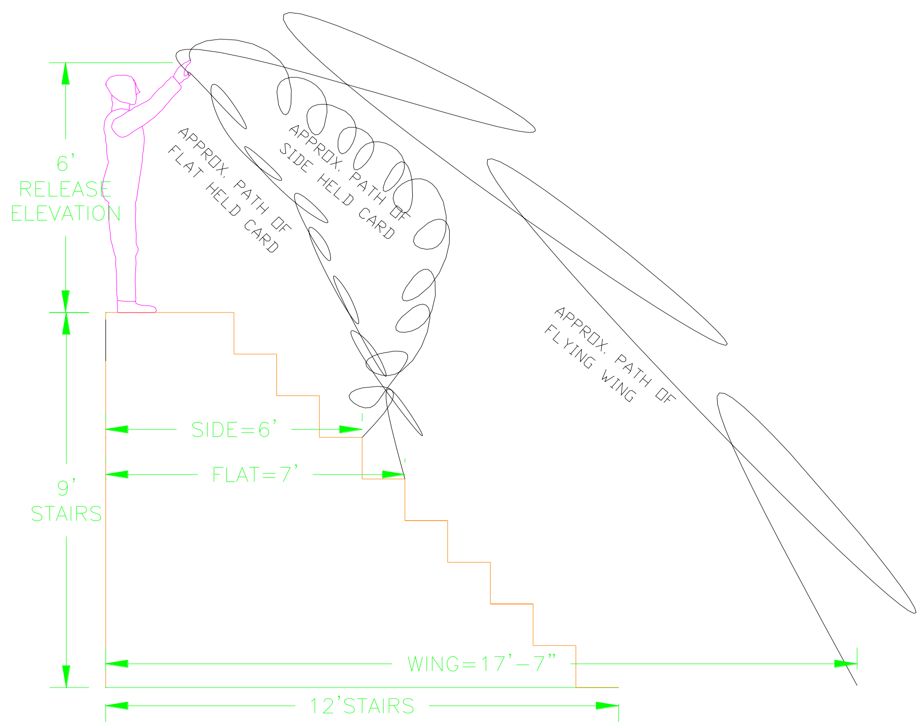

Fig. 1 Courtesy of Scott Little

I held the card in two different positions, one as flat and the other as sideways from my position. Both positions were tested 5 times to obtain some uniformity. All doors and windows near the stairway were closed to minimize cross air currents.

In addition to the business cards, I decided to test the flight patterns of a flying wing. I first saw this design when I was around ten. It was in a paper airplane book that my cousin, who happens to be an engineer, had bought me. I was interested in the fact that this simple folded paper had won a contest in staying aloft over so many more complicated paper airplane designs.

It seemed a good random variable to test, because although it was almost flat, the wing utilized the Bernoulli Principle in its design. All 3 test subjects were held 6 feet above the stairway and let go. The flying wing traveled the farthest at 17’-7”, and the flat card came in second with 7’.

The side card came in last with 6’. The wing could have probably flown farther, but my stairway is only4’ wide and did not allow for much lateral movement. As can be seen, the flat surface has more area to be pushed against and will stay aloft slightly longer.

The position of the cards is shown here in figure 2.

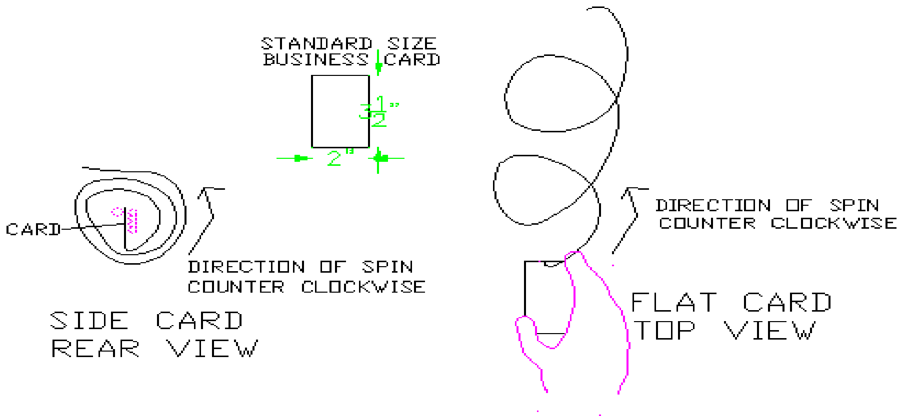

Fig. 2 Courtesy of Scott Little

The flying wing design, as well as an illustration of the Bernoulli Principle, are shown below.

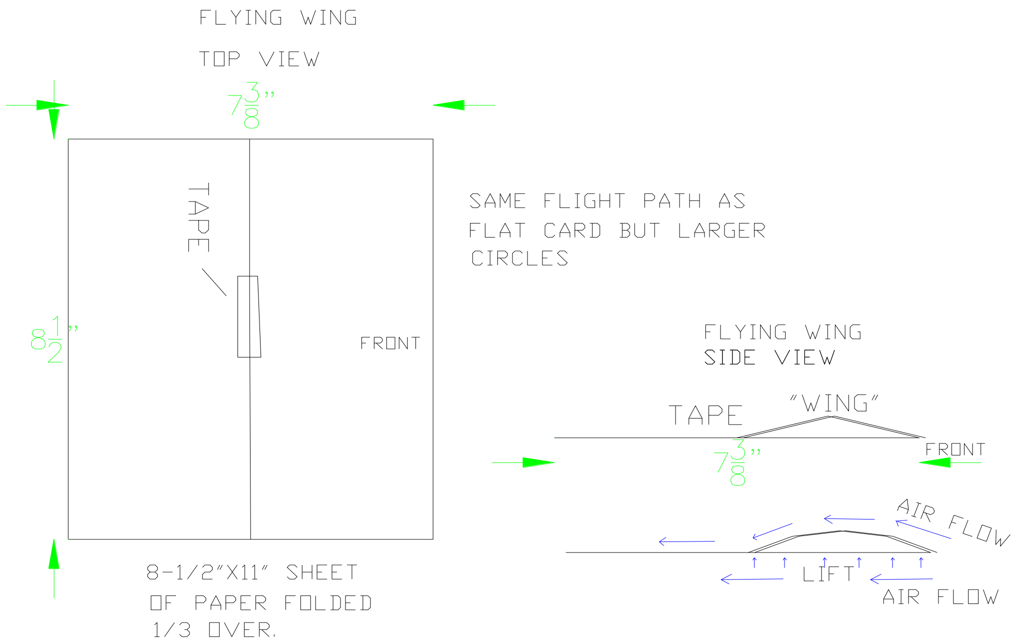

Fig. 3 Courtesy of Scott Little

Although the wing did exhibit the more classical principles of flight, the business card can also stay aloft, but for a shorter period. Perhaps it is combination of all the Bernoulli and Napier-Stokes forces that give the flying wing it's extra lift. But, as Dr. Wang had concluded, a Bumble Bee could not fly by using the same principles a plane uses, so there must other things at work.

By using the Napier-Stokes equations, Dr. Wang concluded that a fluttering piece of paper uses the same physics to stay aloft as a dragonfly's wings. These equations can also explain how a falling leaf can momentarily rise with no wind to carry it aloft. They found a simple way to test her theory; just drop a piece of paper.

In conclusion, there are more forces at work than just one to create lift in a piece of paper. In my personal interpretation of the physics involved, there appears to be a more efficient use of these forces in the flat business card.

This could be due to the larger surface area being exposed in the direction of descent, sort of a "parachute" effect. It could also be that as the flat card fell, it spun in a direction perpendicular to the falling, like a helicopter rotor. The side card spun in the same direction, creating a slight down force. I would like to do more research on these theories, and perhaps plumb the Napier-Stokes Equations for answers.

Sources

(1) Finn, David L " Falling Paper and Flying Business Cards", SIAM News, Vol. 40/ Number 4, May 2007.

(2) http://theory.uwinnipeg.ca/mod_tech/node68.html

(3) http://en.wikipedia.org/wiki/Navier-Stokes_equations